活動 1

進位的計算(Count the dots) - 二進位數字(Binary numbers)

適用年級：國小低年級以上
Age group: Early elementary and up.
預設能力：可以計算到 15 或 31, 並能進行配對和順序
Abilities assumed: Counting up to 15 or 31, matching, sequencing.
所需時間：10-40 分鐘
Time: 10 to 40 minutes.
適用人數：從一個人到全班都適用
Size of group: From individuals to the whole class.

Ø 重要概念(Focus)：

使用 2 為基底來表示數字

Representing numbers in base two.

2 進位的表示模式和相互關係

Patterns and relationships in powers of two.

Ø 摘要說明(Summary)：

現代數位化電腦上所有的資料，幾乎都是採用 0 和 1 的方式來儲存和傳送，這個活動就是要展示如何只以兩個數字〈0 和 1〉來表示所有的文字和數字。

All data in a modern digital computer is ultimately stored and transmitted as a series of zeros and ones. This activity demonstrates how numbers and text can be represented using just these two symbols.

Ø 專有名詞(Technical terms)：

二進位表示法、二進位與十進位的轉換、位元與位元組、字元集

Binary number representation; binary to decimal conversion; bits and bytes; character sets.

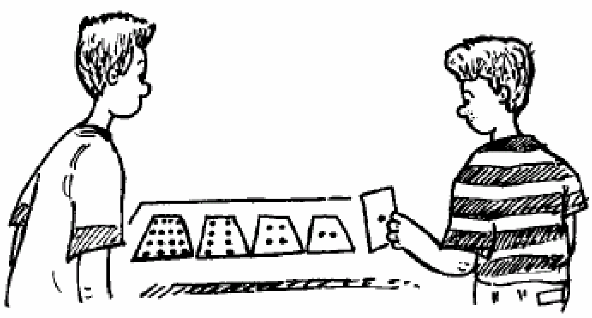

Figure 1.1: Initial layout of the binary cards

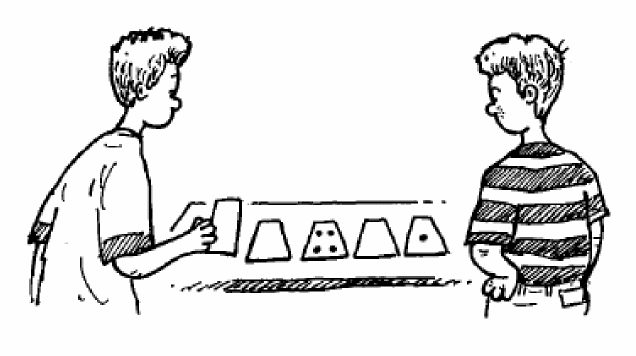

Figure 1.2: Flipping the cards to show five dots

- 所需教材(Materials)：

每一個學生需要：

Each child will need:

從 17 頁中剪下來的一組〈五張〉數字卡

one set of five cards from the blackline master on page 17 (the blackline master has two sets),

18 頁的活動單、

a copy of the black-line master on page 18, and

鉛筆、橡皮擦

a pen or pencil.

- 活動流程(What to do)：

1. 讓學生們座在一起，使他們都可以看到你，發給每位學生一組紙牌
   Seat the children where they can see you, and give each child a set of cards.
2. 讓學生將所有紙牌的圖案向上〈如圖 1.1〉，16 點的紙牌在最左邊，通常會有少部分的學生試圖要排相反的次序，所以一定要檢查一下。對於年齡比較小的學生而言，你可以不要使用 16 點的紙牌。
   The children should lay their cards out, as in Figure 1.1, with the 16-dot card to their left. Some children will be tempted to put the cards in the opposite order, so you should check that they are in descending numeric order from left to right. For younger children, do not use the 16-dot card.

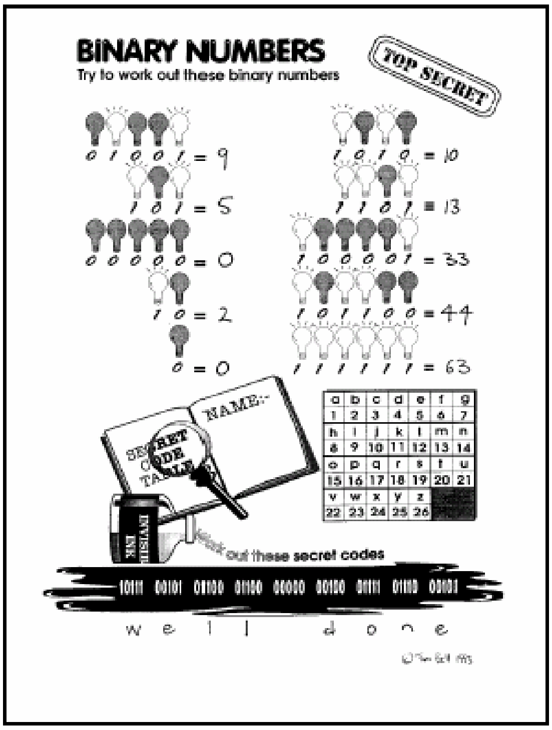

Figure 1.3: Solution to the worksheet on page 18

3. 先讓學生嘗試一下如何翻出總和為 5 點的紙牌，正確的方法是將 1 點和 4 點的紙牌翻過來，其餘的紙牌則保持面朝下（如圖 1.2），每張紙牌不是面朝上、就是面朝下，也就是紙牌上的點數不是全部顯是就是完全沒有。因為總是會出現一些奇特的組合方式來產生 5 點，例如就會有學生拿其他紙牌來遮住 8 點紙牌上的 3 點，這樣似乎也是 5 點。

   Have the children work out which cards to flip over so that exactly five dots are showing. The only (correct) way to do this is to have the 4-dot and 1-dot cards face up, and the rest face down (Figure 1.2). Each card must be either face up or face down, with all or none of its dots showing. Be prepared for some novel ways of getting five dots -it is not unusual for children to produce the requisite number by using spare cards to cover up three of the dots on the eight card!

4. 現在讓學生們其他紙牌上的點數，讓他們試試看，可以組成的點數總共有幾種。

   試試看以下的這些點數，如 3 點（需要 2 點和 1 點的紙牌）、12 點（需要 8 點和 4 點的紙牌）、19 點（需要 16 點、2 點和 1 點的紙牌）等等。對於一些能夠快速且自行完成點數組合的學生，你可以進一步要求他們，是否有其他的方法可以組成這些點數（其實只有一種唯一的方法可以顯示這些數字，但是最好能夠讓他們自己去發現這樣的結果）。

   讓學生討論一下，以目前的紙牌可以表示的最大數為多少？（五張牌是

31 點、四張牌是 15 點）最小數是多少？（學生通常會認為是 1 點，但是正確答案是 0 點）有沒有哪一個數不能被表示出來的嗎？（沒有，所有的數字應該都可以被表示出來，而且都只有一種唯一的表示法）

Now get the children to show other numbers of dots, so that they explore which numbers can be represented.

Ask for numbers such as three (requires cards 2 and 1), twelve (8 and 4), nineteen (16, 2 and 1) and so on. For those who find the combination for a number quickly, ask if they can find another way to get the number (there is only one way to display each number, and they are likely to discover this eventually).

Discuss what the biggest number is that can be made with the cards (it is 31 for five cards, 15 for four cards). The smallest? (Often the number one will be offered first, but the correct answer is zero.) Is there any number between the smallest and largest that can't be represented? (No - all numbers can be represented, and each has a unique representation.)

5. 對於年齡比較大的學生，你可以要求他們依序的排出 1, 2, 3, 4, . . .等數字，並且看看他們是否學會能以一定的次序來增加數字。（其實數字每增加 1，就是將紙牌由右而左的依序翻開）

   For older children, ask them to display the numbers 1, 2, 3, 4, . . . in sequence, and see if they can work out a procedure for incrementing the number of dots displayed on the cards by one (the number of dots increases by one if you flip all cards from right to left until you turn one face up).

6. 這個部分的活動是要使用 0 和 1 來表示紙牌是翻起或覆蓋的狀態，告訴學生我們使用 0 來代表紙牌是隱藏的、1 則代表紙牌是顯示的，例如：圖 1.2 的樣版是 00101，你也可以給其他的範例來試試看，例如：10101 代表 21、11111 代表 31，另外給學生一些練習，讓他們兩者之間可以互轉，你可以讓一部份學生輪流說出自己的生日，並使用 0 和 1 來表示這個日期，而另一部份學生則將這些以 0 和 1 表示的數字轉換成原來的日期，這就是二進位的轉換，也就是以 2 為基底的數字表示法。

   This part of the activity uses zeros and ones to represent whether a card is face up or not. Tell the children that we will use a 0 to show that a card is hidden, and a 1 if its face is showing. For example, the pattern in Figure 1.2 is represented by 00101. Give them some other numbers to work out (e.g. 10101 represents 21, 11111 represents 31). With some practice the children will be able to convert in both directions. You could ask children to take turns calling out the day of the month that they were born on using zeros and ones, and have the rest of the class interpret the date. This representation is called the binary system, also known as base two.

7. 請使用第 18 頁擴充練習上的工作表（完整的工作表請見圖 1.3），這個工作表使用「燈泡」的亮或暗來表示紙牌的隱藏與顯示，燈泡亮表示紙牌顯示，燈泡暗表示紙牌隱藏。前面的幾個題型應該非常容易完成，例如第一題是代表 8 和 1 紙牌顯示，所以代表 9。對於低於五個燈泡的題型，學生應該使用前幾個紙牌即可，例如在第二個題型中，只有使用三個燈泡，從左到右對應的值分別是 4、2 和 1，讓學生試試看是否可以自己完成所有的練習。

   Use the worksheet on page 18 to extend the exercise. (A completed worksheet is shown in Figure 1.3.) The worksheet uses a light bulb that is switched on to represent a card that is showing, and a light bulb that is off to represent a hidden card. The first few patterns should be easy to work out. For example, the first pattern has the 8 and 1 cards showing, so the value represented is 8 + 1 = 9. For the patterns with fewer than five light bulbs, the children should use only the smaller valued cards. For example, the second pattern has only three light bulbs, which correspond (from left to right) to the 4-, 2-, and 1-dot cards respectively. See if the children can work this out for themselves.

   六個燈泡的問題剛好可以配合六張紙牌的問題,每一張紙牌上的點數剛好是前一張的兩倍，點數分別是：1, 2, 4, 8, 16, 32, 64 . . . .，所以 32 點的紙牌剛好可以用來解決六張紙牌的問題。

   The six-bulb questions are designed to make the children think about how many dots should be on a sixth card. The number of dots on each card is double the number on the previous one, so the sequence is 1, 2, 4, 8, 16, 32, 64 . . . . Thus a 32-dot card would be added (at the left) to solve a problem that needs six cards.

   在工作表下面有一個使用 1-26 的數字來表示英文字母的對照表(0 可以用來代表空白)，學生必須先學會每一個數字所代表的英文字，並且能找到對照表中的字母,這個表格代表我們可以將文字的訊息轉換成一系列的 0 與 1，而學生們可以透過這種方法來傳遞編碼過的機密訊息。

   The code at the bottom of the worksheet uses the numbers 1 to 26 to represent the letters of the lphabet. (A zero can be used to represent a space.) The children must work out what each number in the code is, and look up the corresponding letter in the table. This shows how a textual message can be converted to a series of zeros and ones. The children can then write coded messages for each other.

Ø 延伸活動(Variations and extensions)：

在以下的練習中，我們使用棒子的長度來取代紙牌的點數（我們可以使用長

度分別為 1, 2, 4, 8 和 16 單位的棒子來產生代表 0-31 單位的長度），或是使用重量來取代紙牌的點數（我們可以使用重量分別為 1, 2, 4, 8 和 16 單位的東西來產生代表 0-31 單位的重量）

Instead of using cards with dots, the exercise can be done with the lengths of rods (a set of rods of lengths 1, 2, 4, 8 and 16 units can be used to create any length from 0 to 31) or with weights (a set of weights of 1, 2, 4, 8 and 16 units can be combined to produce any weight from 0 to 31).

我們現在可以嘗試使用嗶聲來取代如 01101 的順序，高音頻代表 1、低音頻代表 0，這個活動將會在教室中造成非常吵雜的結果，但是學生們將會印象深刻。其實數據機和傳真機就是使用類似的音頻技術來傳遞資料的，但是持續的高音音頻將會造成類似撕裂的聲音，假如學生不熟悉這樣的聲音，可以讓他們嘗試以電話撥打傳真機，他就可以聽到這樣的聲音。

Instead of calling out sequences like 01101, try using beeps - call out a high-pitched beep for a one and a low-pitched one for a zero. This activity is noisy in the classroom, but children find it memorable! Modems and fax machines use tones like this to transmit information, although the tones are sent so quickly that they blend to make a continuous screeching sound. If the children aren’t familiar with this, they could try calling a fax machine number to hear what it sounds like.

任一個有兩種狀態的物件都可以用來表示數字，圖 1.4 顯示我們可以使用各種不同的方法來來表示數字 9（01001），有一個比較特別的方法是使用手指頭，手指頭向上代表 1、手指頭向下代表 0，我們使用 5 根手指頭可以代表最大的值為 31，10 根手指頭則可以表示最大到 1023，這樣的表示法需要一點點的小技巧，因為它將會產生一些奇怪的姿勢，其實真正的挑戰是使用腳指頭，這將可以讓你表示超過一百萬的數字（真正是多少呢？兩隻手可以表示 0 到 1023，手指和腳指在一起可以表示 1024*1024＝1,048,576）。

Any objects that have two states can be used to represent numbers. Figure 1.4 shows some different ways of representing the number nine (01001). A particularly challenging method is to use your fingers. If a finger is up it represents a one; down represents zero. Counting on your fingers using the binary system enables you to go up to 31 on one hand, and 1023 on two hands.It requires some dexterity, and you have to watch out for rude gestures along the way! For a real challenge, try using your toes as well this will allow you to count up to more than a million. (How many exactly? Two hands give 1024 possibilities, 0 through 1023. Hands and toes give 1024 * 1024 = 1,048576 possibilities, 0 through 1,048,575.)

Figure 1.4: Some unusual ways of representing the number nine (01001 in binary)

中高年級的學生可能對於擴充 1, 2, 4, 8, 16, 32 . .的順序會有極大的興趣，這樣的順序存在著一個非常有趣的關係：假如你將右邊的數字累加到左邊，總和將會永遠比下一個數字少 1。

Older children will enjoy extending the sequence 1, 2, 4, 8, 16, 32 . . . The sequence contains an interesting relationship: if you add the numbers from the beginning from left to right, the sum will always be one less than the next number in the sequence.

二進位有另一個非常有趣的性質那就是：你只要插入一個數字 0 在該數字的最右邊，就會產生 2 倍的效果，例如：1001(9)的倍數是 10010 (18)，中高年級的學生應該可以自己解釋這樣的現象代表什麼（因為所有原來是 1 的位數值現在都是之前值的 2 倍了，所以總和變成原來的 2 倍，相同的效果也會發生在 10 進位上，如果插入一個 0 在該數字的最右邊，就會產生 10 倍的效果）。

Another property of binary numbers is that you can double the number by inserting a zero on the right-hand side of a number. For example, 1001 (9) doubled is 10010 (18). Older children should be able to explain why this happens. (All of the places containing a one are now worth twice their previous value, and so the total number doubles. The same effect occurs in base ten, where inserting a zero on the right of a number multiplies it by ten.)

二進位數字的概念可以被運用在猜數字的遊戲之上，我們可以透過如：「該數字大於或等於 X 嗎？」的問句達成，例如我們知道該數字小於 32，我們可以繼續問「該數字小於 16 嗎？」來逐步達成猜數字的遊戲。活動 5 將有更詳細的描述。

Binary numbers are closely related to the guessing game in which one person thinks of a number and someone else tries to guess it by asking questions of the form “is it greater than or equal to x? ” For example, suppose the number is known to be less than 32. A sensible first question would be “is it less than 16?” The yes/no answers to the questions are given by the zero/one bits in the binary representation of the number. This is explored in detail in Activity 5.

使用 5 個數字可以表示所有的英文字母，但是如果要加上大小寫那就不夠使用了，你可以讓學生算算看電腦到底需要多少不同的字元來表示才足夠（包括小數點、逗點和一些特別的記號如 $ 等），並且結論是要多少字元才能夠儲存所有的字元（兩組英文字母、10 個數字和一些標點符號，全部已經超過了 64 個，所以需要 7 個數字才足夠使用，但是 7 個數字可以表達 128 個字元，已經非常足夠了），目前大多數的電腦內部使用 ASCII (American Standard Code for Information Interchange)來儲存資料，每一個字元使用 7 個位元來表示。

The five-bit code used for letters does not allow both upper- and lower-case letters to be represented. You could have the children work out how many different characters a computer has to represent (including digits, punctuation, and special symbols such as $), and consequently how many bits are needed to store a character. (With two lots of 26 letters, 10 digits, and a few punctuation marks, there are bound to be more than 64 codes needed, so at least seven bits are necessary. Seven bits allows for 128 characters, and this is more than sufficient.) Most current computers use a representation called ASCII (American Standard Code for Information Interchange), which is based on using seven bits per character. Longer codes that allow for the languages of non-English speaking countries are now becoming common.

- 相關知識(What’s it all about?)：

現在的數位電腦幾乎全部都使用上述的方式來表示資訊，這樣的表示方式稱為二進位制，因為只有採用了兩個不同的數字來表示，這也可以稱為以 2 為基底的表示方法（這和一般人所熟知的以 10 為基底完全不同）。每一個 0 或是 1 稱為位元（bit，它是 binary digit 二進位數字的縮寫），「位元」通常被用來表示電腦主記憶體的位址，它是透過電晶體的開或關以及電容器的充電或放電來表示不同的相位。在軟式或硬式磁碟中，位元則是透過磁碟片表面上磁場的方向來表示，不是南-北就是北-南。CD-ROM 則是透過光學的反射來表示，它運用光的反射與否來表示 0 與 1。而當資料要透過電話線或無線電來傳遞時，必須先將資料以高音頻和低音頻來表示 0 或 1。

Modern digital computers almost exclusively use the system described above to represent information. The system is called binary because only two different digits are used. It is also known as base two (as opposed to base ten, which humans normally use). Each zero or one is called a “bit” the term being a contraction of binary digit. A bit is usually represented in a computer’s main memory by a transistor that is switched on or off, or a capacitor that is charged or discharged. On magnetic disks (floppy disks and hard disks), bits are represented by the direction of a magnetic field on a coated surface, either North-South or South-North. CD-ROMs store bits optically the part of the surface corresponding to a bit either does or does not reflect light. When data must be transmitted over a telephone line or radio link, the ones and zeros are commonly represented by high- and low-pitched tones.

一個位元可能無法表示太多資料，所以我們必須將這些位元組合起來使用，我們常以 8 個位元來表示資料，8 個位元可以表示 0-255 的數字，8 個位元我稱之為位元組。

One bit on its own can't represent much, so they are usually grouped together as in the exercise above. It is very common to store bits in groups of eight, which can represent numbers from 0 to 255. A group of eight bits is called a byte.

位元不但可以表示數字，我們也可以拿它來表示文書處理文件中的字元，一個位元通常被用來表示一個單一的文字字元，所以數字 0-255 就可以拿來表示所有的大小寫英文字、數字、標點符號和一些特殊符號。

As well as representing numbers, this code can also represent the characters in a word processor document. A byte is often used to represent a single character in a text - the numbers 0 to 255 are more than enough to encode all the upper- and lower-case letters, digits, punctuation, and many other symbols.

為了要表示更大的數字，我們會將更多位元組合起來使用，兩個位元組（16 位元）就可以被用來表示 65,536 個不同的值，四個位元組（32 位元）可以表示超過 40 億個不同的值。電腦的運算速度會受到它一次可以處理位元多寡的影響，例如 32 位元電腦一次可以進行 32 位元的運算，而 16 位元的電腦因為每次只能進行 16 位元的運算，所以它必須將比較大的數打散成 16 位元的量，所以這樣會造成速度變慢。

To represent larger numbers, several bytes are grouped together. Two bytes (16 bits) can represent 65,536 different values, and four bytes can represent over 4 billion values. The speed of a computer is affected by the number of bits it can process at once. For example, a 32- bit computer can perform arithmetic and manipulations on 32-bit numbers, whereas a 16-bit computer must break large numbers into 16-bit quantities, making it slower.

一般來說，我們無法直接看到電腦上的位元和位元組，因為它們在顯示時已經自動轉換成字元和數字。但是，位元和位元組的觀念，在電腦上用來儲存數字、文字和其他訊息時，仍然是非常重要的一種觀念。

Normally we don't see the bits and bytes in a computer directly because they are automatically converted to characters and numbers when they are displayed, but ultimately bits and bytes are all that a computer uses to store numbers, text, and all other information.

- 延伸閱讀(Further reading)：

大部分的電腦簡介書籍都會討論到二進位的數字系統，在 Gareth Powell 所寫的" My friend Arnold's book of Personal Computers"一書第二章中，有完整的二進位數字的介紹。

Most introductory computing texts discuss the binary number system. My friend Arnold's book of Personal Computers by Gareth Powell has a whole chapter on binary numbers.